

Name and Surname : .....

Teacher : .....

Hudson Park High School

GRADE 11  
MATHEMATICS  
June Examination

Time : 3 hours

Date : 4 June 2013

Marks : 150

Examiner : SLT

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
  2. Number your answers clearly and accurately.
  3. **Start each question at the top of a new side of a page.**
  - 4.1. Fill in the details requested at the top of this page (of the question paper).
  - 4.2. Detach the answer pages from the question paper.
  - 4.3. **NB** Staple your foolscap answers and answer pages together in the correct order.
  - 4.4. **NB** Hand in your answers and question paper separately.
  5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
  6. (Non programmable and non graphical) Calculators may be used, unless their usage is specifically prohibited.
  7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
  8. Geometric statements **MUST** be accompanied by the appropriate geometric REASONS, unless instructed otherwise.
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QUESTION 1 [ 48 marks ]

1.1. Solve for  $x$  :

1.1.1.  $x(2x - 3) = 4$  4

1.1.2.  $-4 \leq 2 - x(x - 5)$  4

1.1.3.  $\frac{x-2}{x-1} = \frac{2x-1}{x+7}$  5

1.1.4.  $-x^{\frac{3}{8}} + x^{\frac{3}{4}} - 6 = 0$  5

1.1.5.  $3\pi x - 7x^{-2} = 0$  3 (21)

1.2. Given :

Graph 1 :  $y = -2x^2 + 4x - 7$

Graph 2 :  $y = -8x + 11$

1.2.1. Calculate the coordinates of the point(s) of intersection of Graph 1 and Graph 2. 6

1.2.2. What can be deduced from your answer to (1.2.1.) ? 1 (7)

1.3. Solve for  $\frac{x}{y}$  :  $3x^2 + 2xy - 8y^2 = 0$  (3)

1.4. Solve for  $x$

• WITHOUT THE USE OF A CALCULATOR

$3^{2x+2} + 3^{x+1} = 1 - 5 \cdot 3^x$  (5)

1.5. Simplify fully

- WITHOUT THE USE OF A CALCULATOR
- answers must be in simplest surd form
- denominators must be rationalised

1.5.1.  $\frac{8}{3\sqrt{32}}$  4

1.5.2.  $\sqrt[3]{\frac{2}{9}} \times \sqrt[3]{\frac{4}{3}}$  2

1.5.3.  $(5\sqrt{2} - 4\sqrt{3})^2$  3

1.5.4.  $\frac{1}{\sqrt{3}-4} - \frac{1}{\sqrt{3}+4}$  3 (12)

QUESTION 2 [ 14 marks ]

2.1. If 3 is a root of

$$-x^2 + 2x = kx^2 - 5k$$

calculate the value of  $k$ . (2)

2.2. Given :  $x = \frac{-2 \pm \sqrt{13-2k}}{3}$

For which values of  $k$  will  $x$  be non-real ? (2)

2.3. Determine the discriminant of the following quadratic equation, whose variable is  $x$  :

$$2x^2 - 6x + 1 = -4kx^2 + 3kx + k^2$$

Multiply out and simplify, your expression for the discriminant. (4)

2.4. The discriminant of a certain quadratic equation, whose variable is  $x$ , is found to be :

$$\Delta = 9k^2 - 12k + 4$$

2.4.1. Factorise the expression for  $\Delta$  (the discriminant). 1

2.4.2. Hence, discuss the nature of the roots of the quadratic equation, whose variable is  $x$ , if  $k \in \mathbb{Q}$ . 5 (6)

QUESTION 3 [ 5 marks ]

3. A tank has two pipes entering it. When operating together, the pipes fill the tank in 40 minutes.

When operating independently, one of the pipes can fill the tank 1 hour faster than the other pipe is able to fill the tank on its own.

How long does it take the faster of the two pipes to fill the tank, if operating on its own ?

**HINT :**  $\text{Rate} = \frac{\text{Volume}}{\text{Time}}$  (5)

QUESTION 4 [ 9 marks ]

4. Given :

Sequence 1 : 50 ; 45 ; 40 ; 35 ; ...

Sequence 2 : 3 ; -10 ; -29 ; -54 ; ...

- 4.1. Determine expressions for the general terms,  $T_n$ , of

4.1.1. Sequence 1 2

4.1.2. Sequence 2 4 (6)

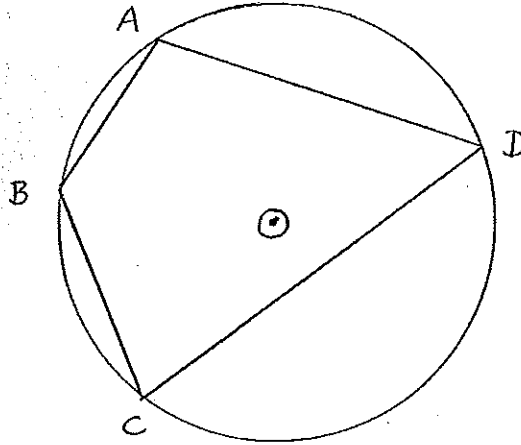
- 4.2. Hence, determine the 91<sup>st</sup> term of the following sequence :

50 ; 3 ; 45 ; -10 ; 40 ; -29 ; 35 ; -54 ; ... (3)

QUESTION 5 [ 13 marks ]

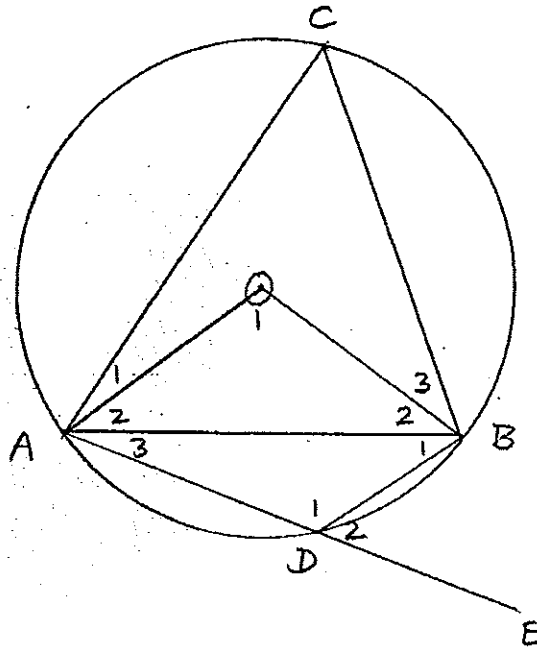
USE THE ANSWER SHEET PROVIDED

5.1. ABCD is a cyclic quadrilateral. O is the centre of the circle.



Prove the theorem which states that :  $\widehat{B} + \widehat{D} = 180^\circ$ . (5)

5.2. O is the centre of the circle and  $\widehat{B}_2 = 40^\circ$ .



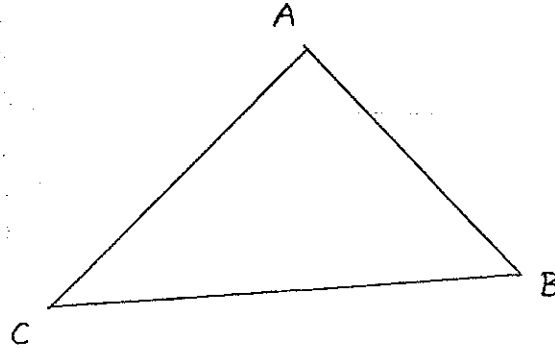
5.2.1. Determine  $\widehat{D}_2$ . 6

5.2.2. If  $AO \parallel DB$ , determine  $\widehat{OAD}$ . 2 (8)

QUESTION 6 [ 8 marks ]

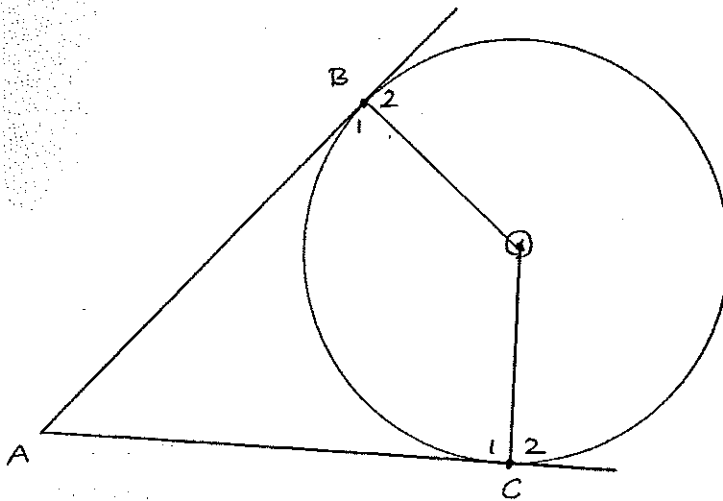
USE THE ANSWER SHEET PROVIDED

- 6.1. In  $\triangle ABC$ ,  $\hat{A} = 4x + 10^\circ$ ,  $\hat{B} = 3x$  and  $\hat{C} = 2x - 10^\circ$ .



Prove that BC is the diameter of the circle passing through A, B and C. (4)

- 6.2. AC and AB are tangents to the circle with centre O.



Prove that ABOC is a cyclic quadrilateral. (4)

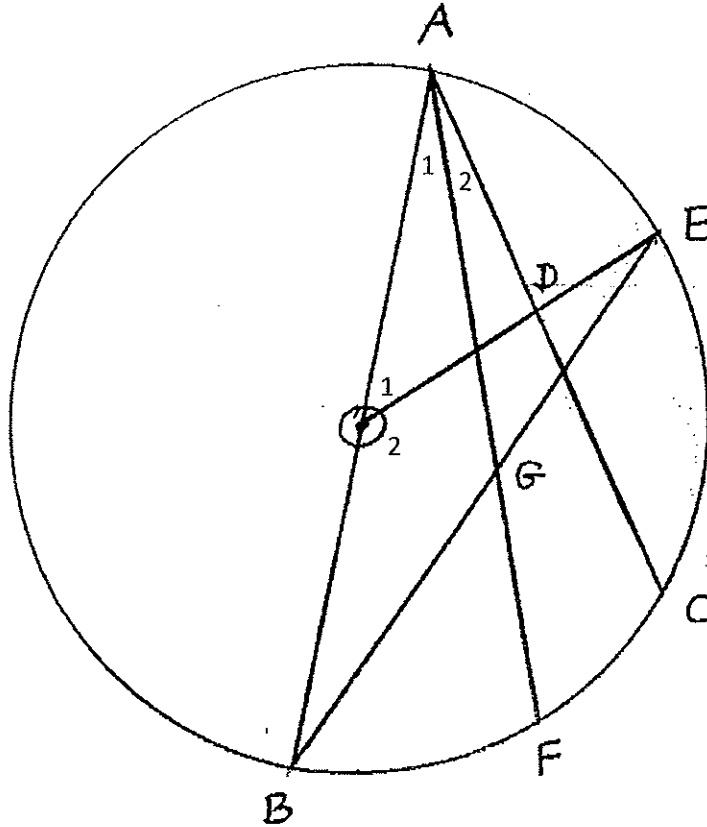




QUESTION 8 [12 marks ]

USE THE ANSWER SHEET PROVIDED

8.  $AOB$  is the diameter of the circle with centre  $O$ .  $DA = DO$  and  $AF$  is the bisector of  $\widehat{BAC}$ .



Prove that :

- 8.1.  $GA = GB$  (5)
- 8.2.  $AOGE$  is a cyclic quadrilateral (3)
- 8.3. If  $C$  and  $E$  are joined, then  $CE \parallel FA$ . (4)

QUESTION 9 [ 8 marks ]

USE THE ANSWER SHEET PROVIDED

9. Given :  $f(x) = -\frac{4}{x+3} - 5$

9.1. Sketch the graph of  $f$ , showing all relevant details on your diagram. ( 5)

9.2. Determine the axis(es) of symmetry of  $g$  if

$$g(x) = f(x) \quad (x > -3) \quad (2)$$

9.3. Determine the equation of  $h$  in  $y$ -form, if  $h$  is the reflection of  $f$  in the horizontal asymptote of  $f$ . ( 1)

QUESTION 10 [ 15 marks ]

10.1 Given :  $f(x) = 2x^2 + 12x + 13$

10.1.1. Write  $f(x)$  in the form  $a(x - p)^2 + q$ , by completing the square. 4

10.1.2. Hence,

10.1.2.1. write down the coordinates of the turning point of  $f(x)$  2

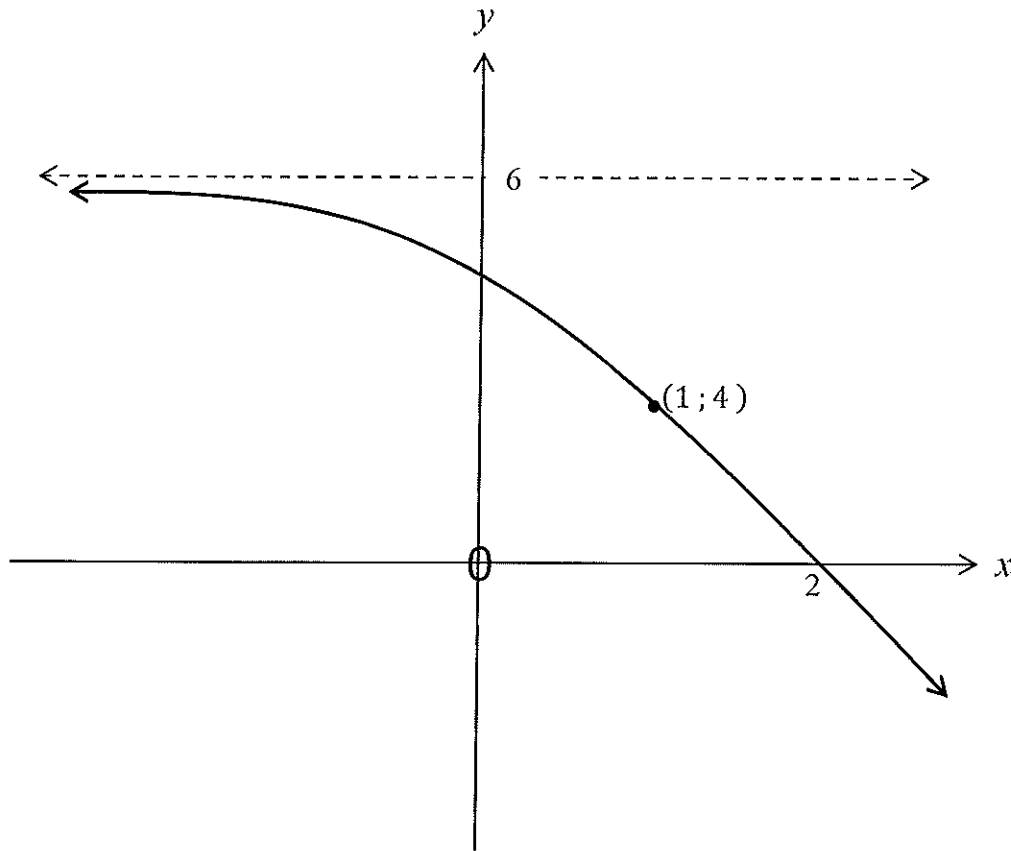
10.1.2.2. calculate the  $x$ -intercepts of  $f(x)$  3 5

10.1.3. Now, if the  $y$ -axis is shifted 2 units to the right, state the new equation of  $f(x)$ , in the form  $y = a(x - x_1)(x - x_2)$ . 2 (11)

10.2. Sketch the graph of  $y = ax^2 + bx + c$ , if  $a < 0$ ,  $b < 0$ ,  $c < 0$  and  $b^2 - 4ac = 0$ . ( 4)

QUESTION 11 [ 8 marks ]

11.  $f(x) = p \cdot q^{x-1} - r$  is sketched below :



11.1. Calculate the values of

11.1.1.  $r$  1

( ) 11.1.2.  $p$  2

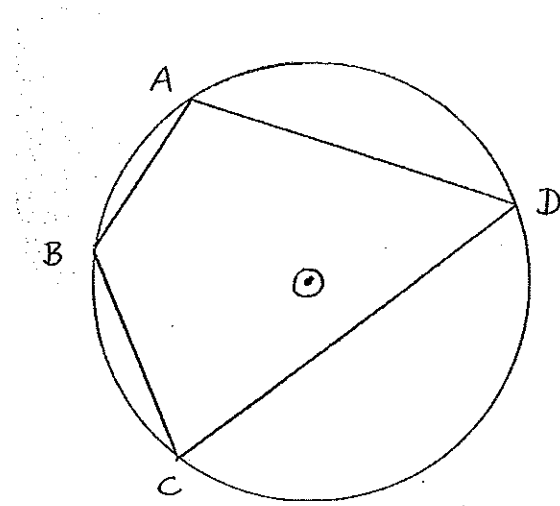
11.1.3.  $q$  2 (5)

11.2. Use the graph to solve for  $x$ , if:  $p \cdot q^{x-1} \geq r$  (1)

11.3. Write down the equation of  $h(x)$  if  $h$  is the reflection of  $f$  in the  $x$ -axis. Leave your answer in  $y$ -form. (2)

ANSWER SHEET FOR QUESTION 5

5.1.



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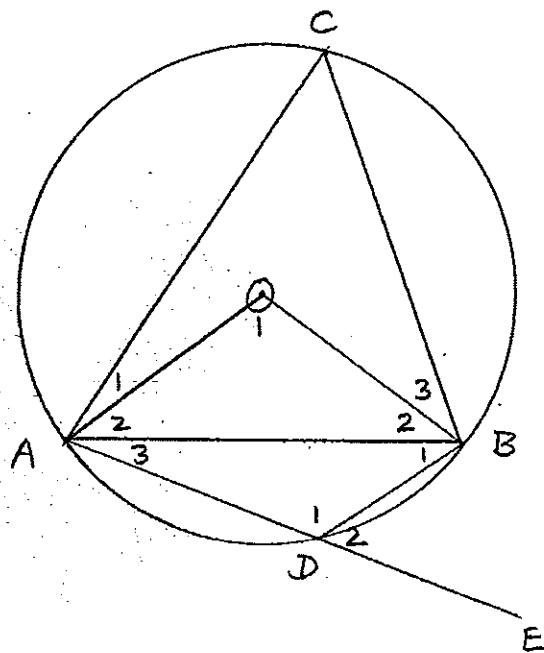
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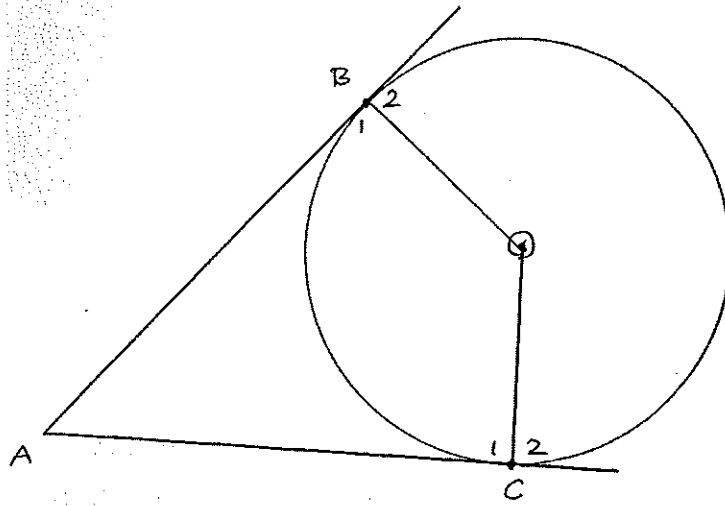
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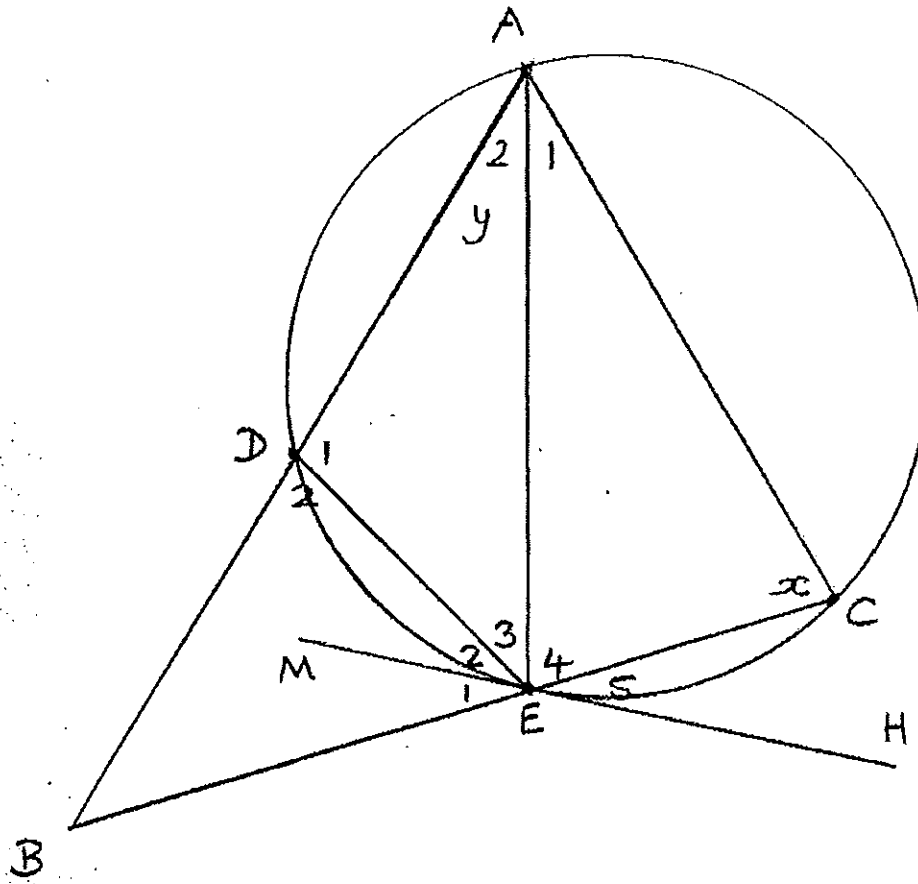
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ANSWER SHEET FOR QUESTION 7

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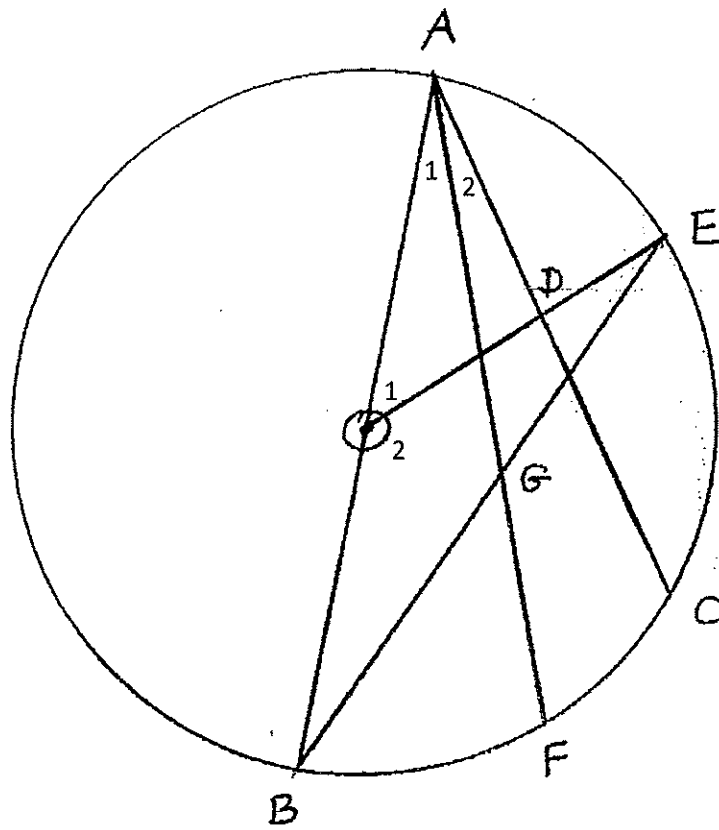


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ANSWER SHEET FOR QUESTION 8

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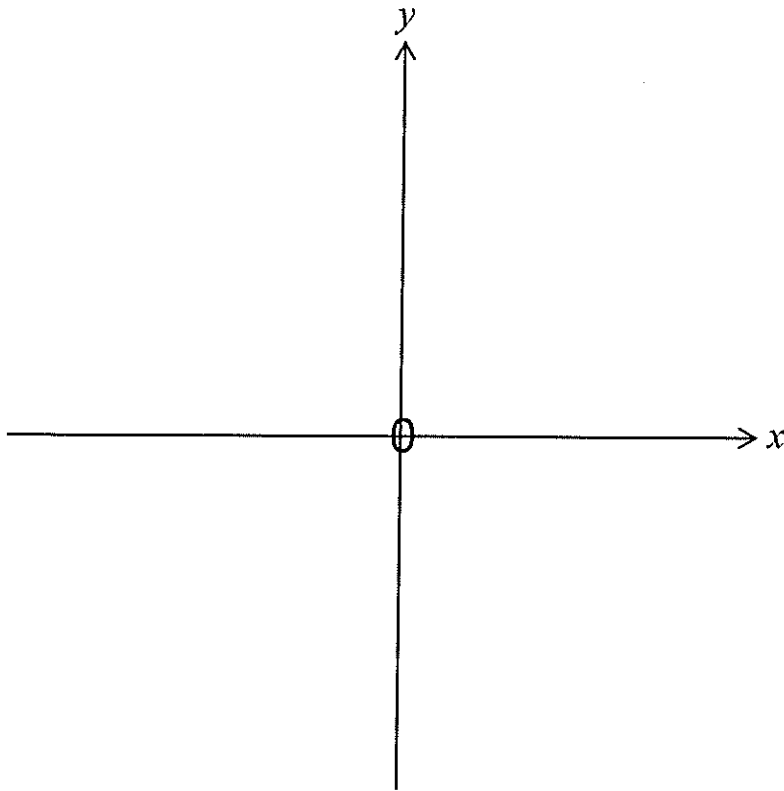
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ANSWER SHEET FOR QUESTION 9

9.1.



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